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Reliability Assessment of Breakwaters

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Reliability Assessment of Breakwaters

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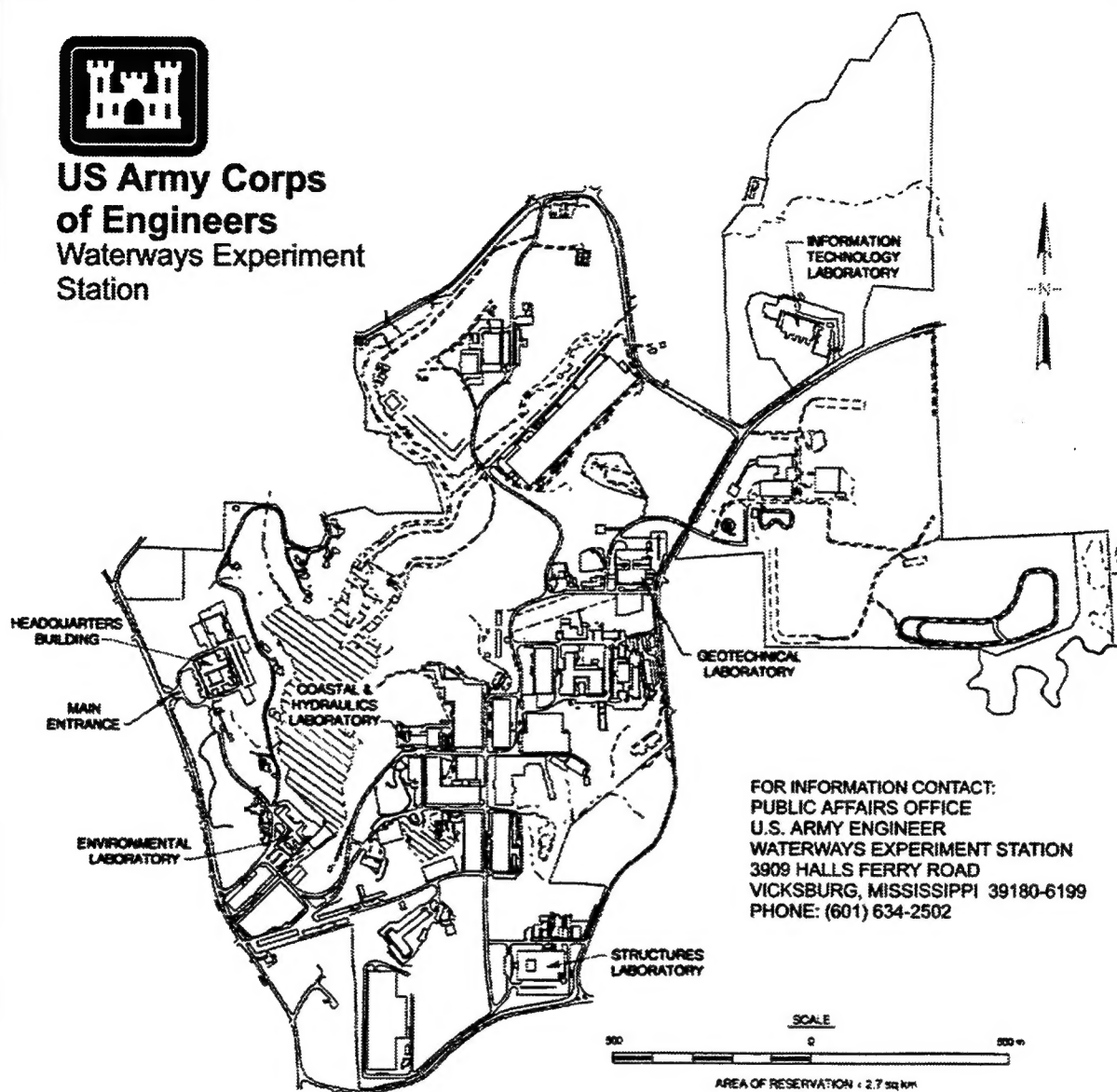
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Preface

Funding for work discussed in this report was provided by the Coastal Navigation and Storm Damage Reduction Program, which is part of the Civil Works Research and Development Program, Headquarters, U.S. Army Corps of Engineers (HQUSACE). The research was funded directly by Work Unit 32792, "Reliability-Based Design and Evaluation of Coastal Rubble Structures" at the Coastal and Hydraulics Laboratory (CHL) of the U.S. Army Engineer Waterways Experiment Station (WES). The CHL was formed in October 1996 with the merger of the WES Coastal Engineering Research Center and Hydraulics Laboratory. Dr. James R. Houston is the Director of the CHL and Messrs. Richard A. Sager and Charles C. Calhoun, Jr., are Assistant Directors.

The Research Coordinator was Mr. David Mathis, of the Directorate of Research and Development, HQUSACE. Messrs. John H. Lockhart, Jr., Barry Holliday, and Charles Chesnutt served as HQUSACE Program Monitors. Ms. Carolyn Holmes, CHL, was the Program Manager. Mr. Jeffrey A. Melby, CHL, was the Principal Investigator of research Work Unit 32792.

The studies were done under the direct supervision of Mr. Gene Chatham, Chief, Wave Dynamics Division, and Mr. D.D. Davidson, Chief, Wave Research Branch, CHL. Mr. Davidson and Dr. Steven A. Hughes, CHL, provided technical review of this report.

At the time of preparation of this report, Dr. Robert W. Whalin was Director of WES and COL Bruce K. Howard, EN, was Commander.

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1 Introduction

The U.S. Army Corps of Engineers (Corps) is responsible for maintaining and constructing coastal structures nationwide. The Corps currently maintains over 1,500 coastal structures. These structures include rubble structures such as breakwaters, jetties, and revetments as well as sea walls, timber cribs, and floating breakwaters. The majority of these structures are of rubble-mound construction. Most of these structures provide protection for navigation projects. They prevent sediment from accumulating in inlets and provide shelter for navigation and for harbors.

The basic deterministic methodology used for designing breakwaters and similar rubble-mound coastal structures was developed several decades ago and includes a combination of analytical techniques, empirical formulae, and physical scale models. Figure 1 shows the primary failure mechanisms that are commonly addressed in modern rubble structure design. The majority of these failure mechanisms are addressed in the design process using empirical formulae. The degree of wave transmission is typically the primary functional criterion.

Preliminary design includes defining project requirements, and then determining environmental design conditions, such as design storm characteristics over the structure's intended life. Structure design follows with specification of structure type and layout, and armor, underlayer, and core material. The structure cross section is determined by specifying the crest height and width, type of toe, and seaward and landward slopes. The crest height and width are the lowest values that will still provide minimum allowable overtopping rates. The type of toe is determined based on the local subgrade material, length of seaside slope, and type of structure armor. The armor is sized to prevent more than a few percent of the units from being displaced in the event of a design level storm. The design process is traditionally deterministic, although some elements of probability theory have been utilized. It has been common practice to specify wave height as a stochastic parameter associated with a design storm with a given return period. But many more stochastic variables must be included in order to adequately assess the reliability of a coastal structure design.

Reliability methods utilize the same probability theory concepts required for specifying a design wave or for determining confidence limits and are therefore quite simple to apply. They are readily adaptable to a wide variety of coastal

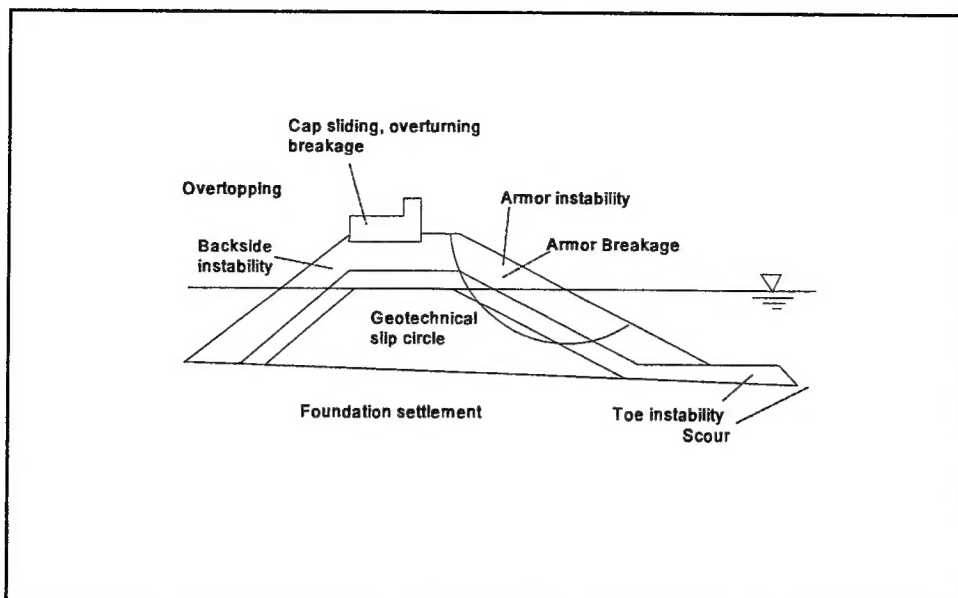


Figure 1. Breakwater failure modes

structure design and evaluation problems and provide a powerful tool for rationally making economic compromises that are always necessary in civil engineering. This report discusses reliability techniques that facilitate inclusion of the random nature of the rubble-mound response in the design process. These reliability techniques can be utilized to gain insight into the degree of uncertainty in the empirical design equations. This is very important for rubble mounds as their response varies widely, even in controlled laboratory conditions. But it must be kept in mind that these methods do not guarantee a more reliable structure. Limited local data, the lack of design guidance for certain failure modes (e.g. scour), and unknown and somewhat unpredictable construction quality, among other things, can reduce a structure's reliability far below the design engineer's estimate. Therefore, while reliability methods provide a powerful tool for gauging and comparing the uncertainty of certain failure modes in our designs, the actual probability of failure of the as-built structure may be significantly different.

Reliability methods can be a powerful tool in estimating uncertainty with respect to many failure modes. Consider, for example, armor stability design. The preliminary design process for specifying the armor layer is deterministic and can be characterized as determining a design load and then defining the appropriate structural capacity. This is done with a single empirical stability equation which incorporates a wave height statistic from an assumed distribution associated with a particular storm return period. The design is then evaluated using scaled physical models. The deterministic design method has several weaknesses, as follows:

- a. Only the loading variables associated with the wave are specified according to measured distributions. The capacity variables are typi-

cally best-fit mean values of widely scattered quantities. Confidence limits are rarely specified with the formulae and provide little help to the designer for determining the risk or reliability in a given design or in the future performance of an existing structure.

- b. Laboratory experiments designed to validate design formulae are done under highly controlled conditions and the prototype may be quite different with more widely varying parameters.
- c. The equations do not include the interaction between failure modes, such as structural response and hydraulic stability or primary armor stability and toe, crown, or transitional armor stability. They typically give no indication of future performance of a damaged structure.

Given the uncertainties in deterministic breakwater design using empirical equations, there is need to incorporate objective evaluation of the performance risks of either new or existing structures. Coastal structures can be allowed to sustain considerably more damage than land-based structures because human lives are typically not in jeopardy. Engineering reliability methods can be used to compare repair scenarios for a given structure or to rank various projects. With repair costs included in the economic analysis, the most economical plan meeting the specified reliability can be selected. This evaluation technique provides more economically efficient coastal rubble structure designs and provides allowance for periodic repairs.

It is standard practice within the Corps of Engineers to use reliability methods to compare competing engineering alternatives. But for coastal structural engineering, little work has been done to synthesize data collected over several decades into a usable format for reliability analysis. Carver (1983) performed a rubble-mound armor stability experiment investigating the variability in stability due to the natural variations in stone placement. As one would expect, the wave height corresponding to the no-damage condition varied widely. Using these data, it is shown in Chapter 3 of this report that the variability in stability can be accounted for in the design process.

Typically for design, all dominant failure modes must be identified and a fault tree constructed for each design alternative. A fault tree is a flowchart of failure modes. An example of a fault tree for a breakwater analysis is shown in Figure 2. The references herein provide guidance on establishing a fault tree. Moritz et al. (1994) performed a reliability analysis of engineering alternatives for a major rehabilitation of the Burns Waterway Harbor, Indiana, breakwater. They describe the fault tree and evaluation of alternatives. Although the reliability calculations are not shown, they provide an overview of the entire alternatives analysis utilizing reliability methods and an economic analysis.

Although several reliability analysis methods have been used for coastal structures, no rigorous comparative analysis of the various methods has been published. Meadowcroft et al. (1996) compared the Level III Monte-Carlo technique to a Level II mean-value approach. Although details of the study were not

described, they found that the Level II approach “provided reasonable agreement” to the more laborious Level III approach. The mean value approach is described in the following two chapters as is a more accurate Level II method termed the “design point approach.”

This report presents an introduction to and comparison of various Level II reliability methods as applied to coastal rubble structure design. Several computational techniques of varying degrees of accuracy are presented. Examples of computational methods are shown in tabular format and FORTRAN programs.

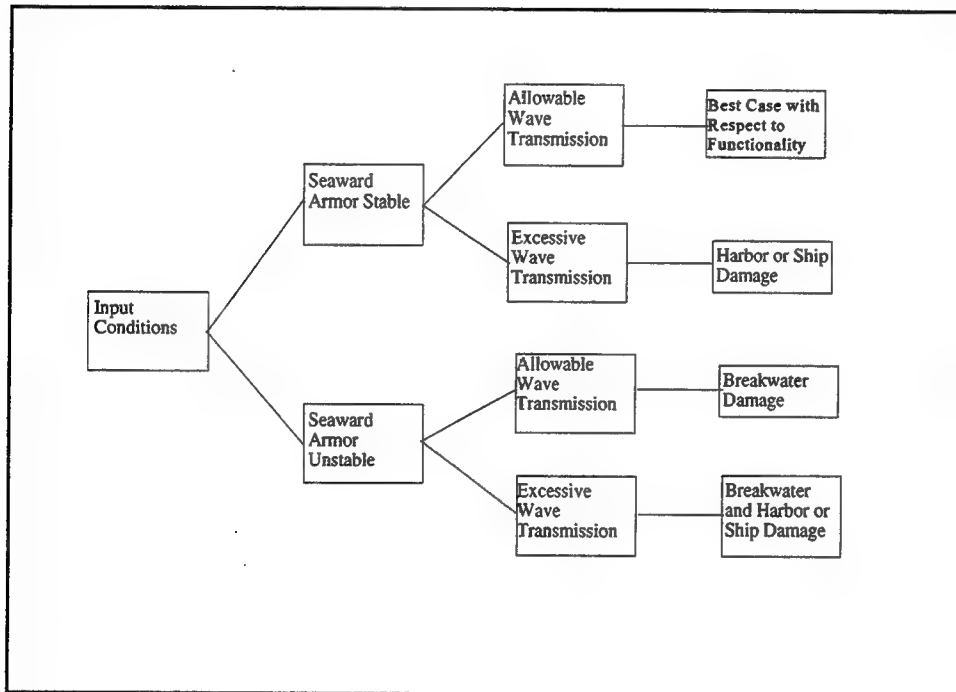


Figure 2. Partial fault tree for breakwater analysis

2 Engineering Reliability Methods

Reliability Methods Overview

Engineering Technical Letter (ETL) 1110-2-532 (U.S. Army Corps of Engineers 1992) provides a review of reliability methods and the underlying probability theory. Other documents that provide good reference are Harr (1987), Benjamin and Cornell (1970), Ang and Tang (1975), and Thoft-Christensen and Baker (1982).

Reliability or conversely, probability of failure, is computed from the probability distribution of a limit state equation. Several reliability definitions that are useful follow.

- a. *Reliability*. Probability that limit state equation will be greater than limit state.
- b. *Probability of Failure*. Probability that limit state equation will be less than limit state.
- c. *Limit State Equation* (otherwise known as the failure function or performance function). Equation describing the engineering performance of interest expressed as either the difference between capacity and demand (safety margin) or ratio of capacity to demand (safety factor). Some authors use the words resistance and load rather than capacity and demand.
- d. *Safety Factor*. Ratio of capacity to demand.
- e. *Safety Margin*. Difference between capacity and demand.
- f. *Limit State*. Level of performance for which capacity equals demand (safety factor = 1 and safety margin = 0).

- g. *Failure Surface.* Surface along the limit state described by the limit state equation.

To illustrate these definitions, consider breakwater armor stability characterized by the Hudson (1958) equation.

$$W = \frac{\gamma_r H^3}{K_D (S_r - 1)^3 \cot \theta} \quad (1)$$

where

W = weight of armor unit

γ_r = specific weight of armor unit material

H = design wave height at the structure toe

S_r = specific gravity of armor unit material

θ = sea-side angle of armor slope relative to the horizontal

K_D = tabulated empirical stability coefficient

For this equation, K_D is defined for a given level of performance, typically the no-damage condition represented by less than 2 percent, by count, of the stones displaced from the seaward structure face. To begin the reliability analysis, the equation of interest is usually rearranged to form a safety factor, $F = C/D$, where C represents capacity and D represents demand. In this case, the limit state is prescribed when the safety factor is equal to one. Performance is satisfactory if the safety factor is greater than one and unsatisfactory when the safety factor is less than one. For a safety factor approach, the Hudson limit state equation could be formulated as

$$F = \frac{W K_D (S_r - 1)^3 \cot \theta}{\gamma_r H^3} \quad (2)$$

Another way to express the limit state equation is as a safety margin, $g = C - D$. The condition under which this margin is less than zero prescribes unsatisfactory performance, the condition above zero satisfactory performance, and the condition at zero describes the limiting state of performance. The Hudson equation can be rewritten in the form of a safety margin as

$$g = W K_D (S_r - 1)^3 \cot \theta - \gamma_r H^3 \quad (3)$$

An equivalent safety margin can also be formulated as

$$g = \Delta D_n (K_D \cot \theta)^{1/3} - H \quad (4)$$

where $D_n = (W/\gamma_r)^{1/3}$ is the nominal diameter of the armor unit and $\Delta = S_r - 1$.

Because some or all of the variables in the limit state equation are nondeterministic, the limit state equation is also nondeterministic. The process of defining the reliability requires defining probability density functions (pdf) of all stochastic variables in the performance function and combining these to determine the overall reliability of the given performance function. The reliability is computed, using probability theory, as the probability that the performance function will exceed the limit state. For the Hudson equation characterized by a safety factor in Equation 2, the reliability is the probability that the weight-dominated capacity parameter in the numerator will exceed the wave loading parameter in the denominator, i.e. the probability that F will exceed one.

Several reliability estimation methods are commonly used in civil engineering to determine the reliability for a particular limit state equation given the pdf's of the underlying variables. These include Level I methods, where a coefficient is used in the design equation to account for uncertainties in the design. This is the method used for the familiar Load and Resistance Factor Design (LRFD) structural design method. Level II methods provide approximations to the reliability, assuming the limit state equation to be normally distributed and then converting all the random correlated non-normally distributed variables to non-correlated normally distributed variables, or assuming a mathematically simplified form of the failure surface, or both. Level III methods utilize the actual distributions of the random variables to compute the reliability of the limit state equation. Specific techniques for the Level II and III methods are discussed below.

- a. *Direct Integration.* A Level III method where the limit state reliability or conversely, the probability of failure, is computed by integrating or convolving the capacity and demand pdf's.
- b. *Monte Carlo Simulations.* A Level III method where the limit state reliability is approximated, through a large number of realizations of capacity and demand pdf's, by the proportion of realizations where the limit state equation is greater than the limit state.
- c. *Taylor Series Approximations.* A Level II method where the limit state equation is approximated by a Taylor Series expansion about some critical point usually truncated to first or second order. The reliability is computed as the minimum distance between the failure surface and zero.
- d. *N-Point Estimates.* A Level II method where the pdf of the limit state equation is approximated as N lumped masses.

Taylor series and N-Point estimates typically provide reasonable accuracy without having to perform thousands of calculations. They are particularly useful when data are sparse and use of empirical equations derived from data entails a great deal of uncertainty. Such is typically the case with coastal structure design. Level II methods also provide insight into the sensitivity of reliability to various parameters throughout the calculation process that is useful for the engineer. Several popular Level II methods are compared herein. More accurate Level III methods can be used to determine the reliability if the density functions of all nondeterministic variables are known or assumed; but convolving more than two density functions is computationally intensive and is therefore not generally practical. Also, for coastal structure engineering, the probability distributions of the various parameters are seldom known with any certainty.

Reliability Concepts

As stated above, the first step to estimate the reliability of a design is to establish a limit state equation in one of two forms:

$$\text{safety margin: } g(\tilde{x}) = C(\tilde{x}_C) - D(\tilde{x}_D) \quad (5)$$

$$\text{safety factor: } F(\tilde{x}) = C(\tilde{x}_C)/D(\tilde{x}_D) \quad (6)$$

such that $g > 0$ or $F > 1$ represents satisfactory performance. In these equations, \tilde{x} represents a set of stochastic variables describing geometry, material properties, and loading for the particular limit state, C is a function of the set of stochastic capacity variables \tilde{x}_C , and D is a function of the set of stochastic demand variables \tilde{x}_D . Accordingly, the limit state function is also stochastic. Consequently g and F can be described by pdf's, as illustrated in Figure 3. Note that g or F can be a function of many variables. The failure surface is defined by the limit state $g=0$ or $F=1$. If the limit state is a function of more than two variables, say N variables, an imaginary failure "surface" in N -dimensional hyperspace can be described.

Reliability with respect to the performance function g can be computed from the relation $R=P(g>0)$, which states that reliability R is the probability of satisfactory performance ($g>0$). P represents the exceedance probability distribution of g . Conversely, the probability of failure P_f with respect to the performance function is given by $P_f = P(g<0)$. Similarly, for a safety factor representation $R=P(F>1)$ and $P_f = P(F<1)$. Figure 3 illustrates several of these points. The region under the pdf to the left of the origin represents the probability of failure and that to the right represents the reliability. Note that the term *failure* is not used to describe the structure's performance directly. It is used to describe the fact that the performance function does not meet the design criteria. For exam-

ple, if the performance function for armor stability, say the Hudson equation, describes the design condition for "2-percent armor displacement," then the term *failure* would describe the limiting condition of 2 percent of the armor being displaced. This contrasts with the conceptual picture most of us might have of breakwater failure where the rubble mound has collapsed or been breached.

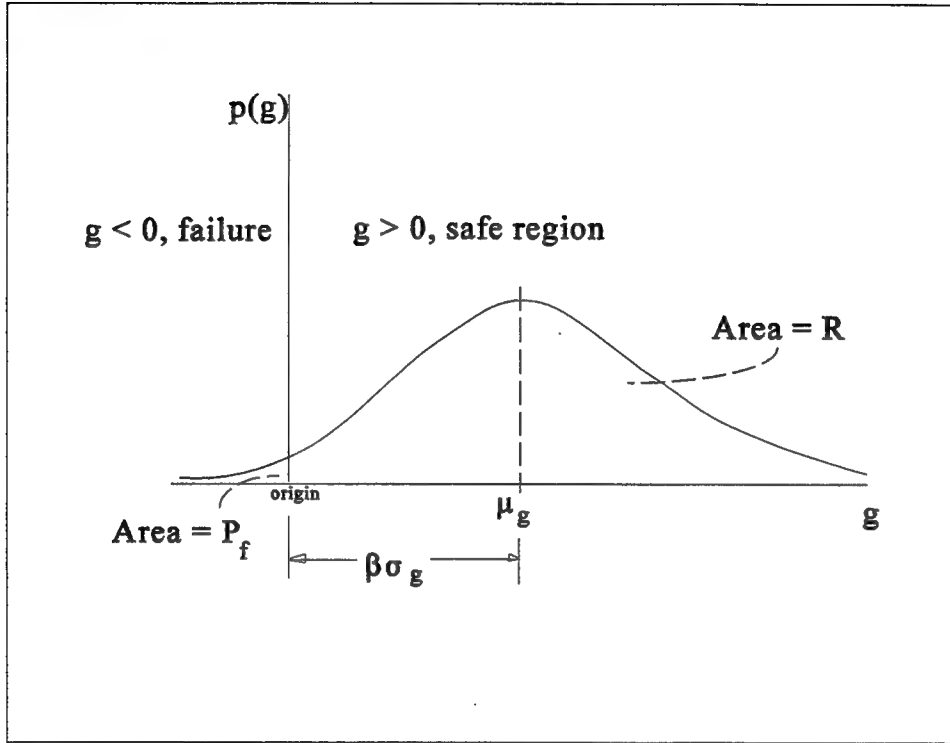


Figure 3. Definition sketch for the limit state equation pdf and reliability index

Often the limit state equation pdf is not completely known but the mean and standard deviation are known. Based on the principle of maximum entropy, the normal distribution can be assumed if only the first two moments are known (Harr 1987). If capacity and demand functions are assumed to be normally distributed, then the limit state relation will also be normally distributed. If capacity and demand are expressed in standard normal form

$$C' = \frac{C - \mu_C}{\sigma_C} \quad D' = \frac{D - \mu_D}{\sigma_D} \quad (7)$$

then the limit state relation ($g = C - D = 0$) becomes

$$\sigma_C C' - \sigma_D D' + \mu_C - \mu_D = 0 \quad (8)$$

where μ_C and σ_C are the mean and standard deviation of the capacity function and μ_D and σ_D are the mean and standard deviation of the demand function. Note $g(C,D) = 0$ is displaced a distance β from the origin in the transformation to $g(C',D') = 0$. This distance can be found from simple geometry (Hasofer and Lind 1974) as

$$\beta = \frac{\mu_g}{\sigma_g} = \frac{\mu_C - \mu_D}{\sqrt{\sigma_C^2 + \sigma_D^2}} \quad (9)$$

if C and D are uncorrelated (Figure 4). β is termed the *reliability index*. Here μ_g and σ_g are the mean and standard deviation of the safety margin performance function. If C and D are correlated, the expression becomes

$$\beta = \frac{\mu_g}{\sigma_g} = \frac{\mu_C - \mu_D}{\sqrt{\sigma_C^2 + \sigma_D^2 - 2\rho_{CD}\sigma_C\sigma_D}} \quad (10)$$

where ρ_{CD} is the correlation coefficient between C and D defined as

$$\rho_{CD} = \frac{\sigma_{CD}}{\sigma_C\sigma_D} = \frac{E[(C - \mu_C)(D - \mu_D)]}{\sigma_C\sigma_D} \quad (11)$$

where σ_{CD} is the covariance between C and D and $E[]$ is the expected value of the quantity inside the brackets. Note that β is simply a unit standard normal variate along $\tilde{z} = (\bar{x} - \mu_x)/\sigma_x = 0$. The reliability can therefore be approximated using the reliability index as

$$R = P(g > 0) = \int_0^{\infty} p_g(x) dx = \Phi(\beta) \quad (12)$$

where $p_g(x)$ is the pdf of $g(x)$ and $\Phi(\beta)$ is the widely tabulated standard normal distribution function evaluated at β . The definitions for $F(x)$ would be similar; but the log-normal distribution would be used because the normal distribution is an appropriate approximation for sums of variables while the log-normal is appropriate for products.

The reliability index is graphically defined in Figures 3 and 4. It can be seen that the index represents the distance from the expected value of the performance function to the failure surface in units of standard deviation. Thus $\beta = 2$ implies that the mean of the performance function lies two standard deviations

above the limit state. The reliability index is then a measure of the likelihood of the structure to perform satisfactorily, with respect to the performance function in question. Reliability increases with increasing mean and with decreasing standard deviation of the performance function. Table 1 shows the correspondence between the reliability and the reliability index. This correspondence will be valuable later when interpreting applications of the reliability methods. In Table 1, Φ^{-1} is the inverse of the normal distribution function.

Table 1
Correspondence Between Reliability Index and Reliability

$R = P(g > 0) = P(F > 1)$	$\beta = \Phi^{-1}(R)$
0.500	0.0
0.690	0.5
0.840	1.0
0.933	1.5
0.977	2.0
0.9987	3.0
0.999968	4.0

In general, g will be a nonlinear function of the random variables \bar{z} . The case for two independent random variables is depicted in Figure 4. Note that the transformation into a normalized coordinate system has been performed where $\bar{z} = (\bar{x} - \mu_x)/\sigma_x$. The reliability index is the minimum distance from the mean state $\bar{z} = 0$ to the limit state $g(\bar{z}) = 0$. The closest point on $g(\bar{z}) = 0$ is known as the design point z_d .

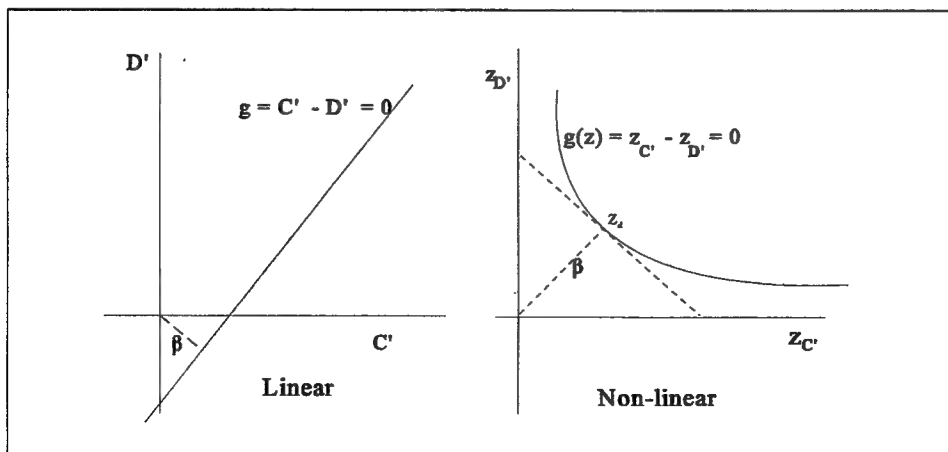


Figure 4. Example sketch of linear and nonlinear limit states

Often in computations, the limit state relation is approximated as a linear failure surface using a Taylor series expansion either about the mean point or about the design point. An important characteristic of the design point expansion is that the resulting reliability index is invariant to the arbitrary functional form of the limit state expression; whereas a mean point approximation will be dependent on the functional form of the limit state. The two methods will be compared in the next section.

If the random variables \tilde{z} are non-normal, an approximate normal distribution can be fitted to the actual distribution such that the density and distribution are unchanged at the design point. With the design point normal variate designated $z_d = (x_d - \mu_x)/\sigma_x$, the mean and standard deviation of the fitted density function are

$$\begin{aligned}\sigma'_x &= \frac{\phi[\Phi^{-1}(P_x(x_d))]}{p_x(x_d)} \\ \mu'_x &= x_d - \Phi^{-1}(P_x(x_d))\sigma'_x\end{aligned}\quad (13)$$

where P and p are the distribution and density functions of random variable x , Φ and ϕ are standard normal distribution and density functions, and Φ^{-1} is the inverse of the standard normal distribution.

Taylor Series Approximations of Reliability

Taylor series mean point method

Based on the preceding discussion it appears that a simple method for estimating the reliability would be to approximate the limit state either at the mean or design point on the failure surface. This can be done using a Taylor series expansion of the function about either point. The mean point expansion can be done directly; but, because the design point is a priori unknown, the design point solution requires an iterative approach. Either way, g is first linearized about the mean point $\tilde{x} = 0$. The Taylor series approximation of the safety margin about the mean point is given by

$$g \doteq g(\mu_{\tilde{x}}) + \nabla g \bullet (\tilde{x} - \mu_{\tilde{x}}) \quad (14)$$

or in scalar notation

$$g \doteq g(\mu_{\tilde{x}}) + \sum_{i=1}^N \frac{\partial g}{\partial x_i} (x_i - \mu_{x_i}) \quad (15)$$

where \doteq is used to denote a first order approximation, $\nabla g \bullet$ is the dot product operation using the gradient of g , partial derivatives are evaluated at each mean value, and N is the number of random variables in g . The general expressions for the expected value and variance of g as a function of correlated random variables are

$$E[g] = \mu_g \doteq E[g(\mu_{\bar{x}}) + \nabla g \bullet (\bar{x} - \mu_{\bar{x}})] = g(\mu_{\bar{x}}) \quad (16)$$

$$V[g] = \sigma_g^2 \doteq \nabla g V[\bar{x}] \nabla g^T \quad (17)$$

where E is expected value and V is variance and ∇g^T is the transpose of ∇g . For N variables, the variance becomes

$$\sigma_g^2 \doteq \sum_{i=1}^N \sum_{j=1}^N \left(\frac{\partial g}{\partial x_i} \frac{\partial g}{\partial x_j} \sigma_{x_i x_j} \right) \quad (18)$$

and, if the variables are uncorrelated, the variance is

$$\sigma_g^2 \doteq \sum_{i=1}^N \left(\frac{\partial g}{\partial x_i} \sigma_{x_i} \right)^2 \quad (19)$$

The reliability index and reliability can be computed from the mean and variance using equations 10 and 12, respectively.

For the safety factor solution, the expected value and variance can be computed using the log of the mean and variance of the safety factor relation approximated at the mean or design points according to Benjamin and Cornell (1970) as

$$\mu_{\ln F} = \ln \mu_F - \frac{\sigma_{\ln F}^2}{2} \quad (20)$$

$$\sigma_{\ln F}^2 = \ln \left(\left(\frac{\sigma_F}{\mu_F} \right)^2 + 1 \right) \quad (21)$$

where $\mu_F \doteq F(\mu_x)$, as in Equation 16, and σ_F is similar to Equation 18. Then the reliability index is $\beta = \mu_{\ln F} / \sigma_{\ln F}$ and reliability $R = \Phi(\beta)$.

Taylor series finite difference approximation

The Taylor series mean point approximation is typically employed using a finite difference approximation for the partial derivatives in the variance. The calculation is often performed using the difference between the limit state relation at one standard deviation above the mean and one standard deviation below the mean as follows

$$\frac{\partial g}{\partial x_i} = \frac{g(x_i + \sigma_i) - g(x_i - \sigma_i)}{2\sigma_i} \quad (22)$$

This equation is often written in shorthand as

$$\frac{\partial g}{\partial x_i} = \frac{g^+ - g^-}{2\sigma_i} \quad (23)$$

The safety factor equations would be similar. The mean point analysis with finite differencing is commonly known as the *Taylor Series Finite Difference* (TSFD) method. The strength of this technique of reliability estimation is that it provides a closed form solution that can be done easily using a handheld calculator or a spreadsheet program. The weakness of the mean point method is that the solution is dependent on the form of the limit state equation, i.e. β and R can vary depending on the form chosen. Additionally it is less accurate than more rigorous solutions. Example calculations are shown in Chapter 3.

Taylor series design point method

Mlakar (1994) proposed a more accurate iterative solution for the design point definition of the reliability index. Consider $\tilde{z}_d = \lambda \nabla g(\tilde{z})$ as an iteratively improved estimate of the design point, where λ is an arbitrary constant. Then substituting into Equation 14, expressed as a function of \tilde{z} , yields the design point on the linearized failure surface which is closest to the mean as

$$\tilde{z}_d = \frac{\tilde{z} \cdot \nabla g(\tilde{z}) - g(\tilde{z})}{\nabla g(\tilde{z}) \cdot \nabla g(\tilde{z})} \nabla g(\tilde{z}) \quad (24)$$

$\beta_d = |\tilde{z}_d|$ now represents an estimate of the reliability index. This solution is similar to TSFD but more generalized. The iterations start with a mean point approximation. At each iteration step, a new design point is computed from Equation 24 and the iterations continue until convergence is achieved. Typically the partial derivatives are numerically approximated using a finite difference formulation such as

$$\frac{\partial g}{\partial z} = \frac{g(z+\Delta) - g(z)}{\Delta} \quad (25)$$

where Δ is some small increment in z . Hereafter, this iterative TSFD method will be referred to as ITSFD.

A general FORTRAN program for the ITSFD computation is shown in Appendix A. The main program comprises the reliability engine, while the specific limit state equation is computed in subroutine LIMSTATE. Subroutine DEP_MEAN is used to compute a range of one variable in the limit state equation so a range of reliabilities can be computed at one time. An example break-water limit state equation is encoded in Appendix A and example computations are discussed in Chapter 3.

Point Estimate Method

Point estimate methods (PEM), as described in Harr (1987) and ETL 1110-2-532 (USACE 1992) provide invariant approximations to the reliability index. The N -point estimates proceed in a similar fashion to the Taylor series approximations except, rather than the limit state relation being approximated, the probability distributions are approximated at N points. PEMs have the advantage that they do not require computation of derivatives of the limit state equation nor do they impose assumptions on the existence and continuity of the first few derivatives of the limit state equation as do the Taylor series expansions (Rosenbluth 1975). They also require no assumptions on the correlation of independent variables. Further, the methods can be done on a calculator or in a spreadsheet and do not require the laborious calculations of the Monte Carlo method. Accuracy of the methods depends primarily on the accuracy of the approximate probability mass functions.

A common PEM is a two-point approximation to the probability distribution, lumping the distribution into two masses at one standard deviation above and below the mean. This method is similar to the TSFD mean point technique described above. The accuracy of the method is similar to the accuracy of the TSFD method discussed above.

System Reliability

The reliability of various modes of failure or multiple components in a design can be treated using a system analysis. System analyses are discussed in the references and will only briefly be discussed herein. System reliability is determined by combining the components in series or parallel fashion, or a combination of the two.

Series system

A system of components can be considered to be in series if failure of any one component leads directly to system failure. If an individual component has a probability of satisfactory performance, or reliability, of R_i , then the reliability of a system of n components in series will be the product of the individual reliabilities as

$$R_s = R_1 R_2 R_3 \dots R_i \dots R_n \quad (26)$$

Parallel system

A system can be considered to be parallel if failure of all components must occur for system failure to occur. Therefore the reliability of a parallel system is given by

$$R_s = 1 - (1 - R_1)(1 - R_2)(1 - R_3) \dots (1 - R_i) \dots (1 - R_n) \quad (27)$$

Parallel series system

Most projects can be considered to be a collection of parallel and series systems. The analysis becomes very complex when interaction between the components is considered. This is particularly true for coastal structures, as the interaction between modes of failure is generally unknown. Several methods exist for determining the reliability of a complex system. The most widely used is simply to determine the upper and lower bounds using a separate series and parallel analyses. Harr (1987) suggests methods for narrowing the bounds. Another common technique for analyzing complex systems is to apply weighting factors to each component. These methods are beyond the scope of this report but are discussed at length in the references.

The most practical approach may be to focus on the components or subsystems that govern the reliability of the overall system (USACE 1992). Therefore, breakwater reliability may be judged acceptable if the reliability index for each component exceeds a specified level.

3 Breakwater Reliability Examples

Introduction

In this chapter the reliability computational methods of the previous chapter are applied to coastal rubble structure design, and the methods are compared. The reliability indices are computed for common limiting states of performance including armor unit stability, armor structural response, and runup and overtopping. Any limit state equations or performance functions can be used in the reliability analysis. Those used herein are considered accepted practice as established in the *Shore Protection Manual* (SPM 1984). In addition to the Hudson equation, as previously described, the stability equations of van der Meer (1987) are also evaluated. The performance functions of SPM sections 7-II-1 and 7-II-2 are used for runup and overtopping, respectively. The methods of Melby (1989, 1993) are used for concrete armor stress evaluation.

Hudson Armor Stability Limit State

The Hudson equation was rewritten in the form of a safety factor (Equation 2) or as a safety margin (Equations 3 and 4) in Chapter 2. All variables in the Hudson equation are considered stochastic in this analysis.

Wave height

The single-storm wave heights generally follow a Rayleigh probability distribution of the form

$$p(\zeta) = 2\zeta e^{-\zeta^2} \quad (28)$$

where $\zeta = H/H_{rms}$ and H_{rms} is the root mean square wave height. The significant wave height is usually given in design and can be incorporated using the relation $H_s = 1.416 H_{rms}$. The mean and standard deviation required for the reliability analysis are given by $\mu_H = 0.886H_{rms}$ and $\sigma_H = 0.463H_{rms}$, respectively.

Note that any distribution of wave heights could be used, including a measured distribution or an extremal Weibull distribution. Additionally, because water depth and wave height distributions are correlated in shallow water, any long-term analysis must take this into account. A number of techniques exist to facilitate combination of the long-term wave height and water level distributions, including Monte-Carlo simulation, Empirical Simulation Technique, (USACE 1996), and direct integration of historical distributions. Within this report, the wave height distribution is overly simplified to focus on the reliability calculation.

Empirical stability coefficient

The stability coefficient depends on many factors including armor shape and roughness, wave period, depth, storm duration, level of damage, and offshore bathymetry. In deterministic design, the minimum stability coefficient derived from hydraulic stability tests is typically used. For example, for two layers of rough angular stone placed on a structure trunk exposed to breaking waves, the SPM recommends a minimum stability coefficient of 2. But, as can be seen in Figure 5, there is a wide variation of minimum stability coefficients, even for the limiting case of breaking waves on a breakwater trunk. The variation shown in this figure is only due to variability in laboratory mound construction and variability in wave phasing. Carver (1983) provided data which suggest a mean K_D of 2.59 and a standard deviation of 0.65 for angular stone on a jetty trunk exposed to breaking waves. For concrete armor units, the variation of stability coefficients is equally wide. For dolos, data of Carver (1983) suggest a mean of 21.5 and standard deviation of 5.6 for similar wave and structure conditions. For dolos stability, the coefficient of variation, defined as the ratio of the standard deviation to the mean, is 0.26, according to these data. For other concrete armor shapes, a similar coefficient of variation can be assumed where insufficient data exist to estimate the value.

Structure seaside slope, armor specific weight, and armor weight

The seaward structure slope θ is usually carefully controlled in construction and, with no supporting data, a coefficient of variation of 0.05 is provisionally used herein. The specific weight of stone and concrete γ_r is assumed to have a coefficient of variation of 0.03 based on prototype data (Bryant and Mlakar 1990). The final stochastic variable is the armor weight. For stone, the gradation is prescribed as 75 percent to 125 percent and can be assumed provisionally to follow a normal distribution with a coefficient of variation of 0.10. For concrete armor, the coefficient of variation of weight is small and is assumed to be 0.01.

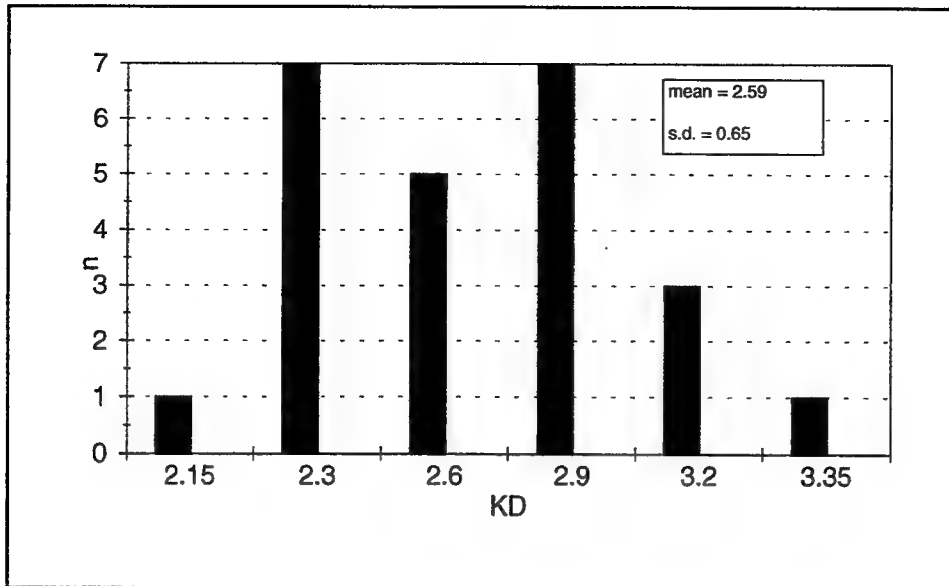


Figure 5. Histogram of Hudson stability coefficients for stone armor on breakwater trunk exposed to breaking waves (Carver 1983)

Table 2 summarizes the variable means and standard deviations for angular stone and dolosse used in the following reliability analysis.

Table 2 Statistics for Hudson Limit State Variables			
Random Variable	Mean	Standard Deviation	Coefficient of Variation
	μ	σ	c
Stone weight, W	Range	Range	0.10
Stone eq. coefficient, K_D	2.59	0.65	0.25
Stone specific weight, γ_r	2405 kg/m ³	72 kg/m ³	0.03
Dolos weight, W	Range	Range	0.0
Dolos eq. coefficient, K_D	21.5	5.6	0.26
Dolos specific weight, γ_r	2405 kg/m ³	72 kg/m ³	0.03
Wave height, H	2.70 m	1.41 m	0.52
Structure slope, $\cot\theta$	2.0	0.1	0.05

Iterative TSFD reliability calculation

The reliability calculation using the ITSFD approach of Equation 24 and the safety margin Hudson limit state of Equation 3 has been applied herein using the statistics from Table 2. The resulting reliability and reliability indices versus the expected value of weight of stone are shown in Figure 6. A similar plot is shown for dolos armor in Figure 7. In these figures, B refers to β , the reliability index. In Figure 6, the results for the equations of van der Meer (1985) indicated by VdM are also shown. These equations will be discussed in the following section. As can be seen in Figures 6 and 7, reliability increases with armor weight from zero quickly, approaching $R = 1$ asymptotically. Using the SPM recommended $K_D = 2$ with $H_{1/10} = 1.8$ $H_{rms} = 5.5$ m yields a design armor weight of 41 tons, which has a reliability of $R \approx 0.98$. The corresponding reliability index is approximately 2. The reliability index increases at a slower rate than R with armor size.

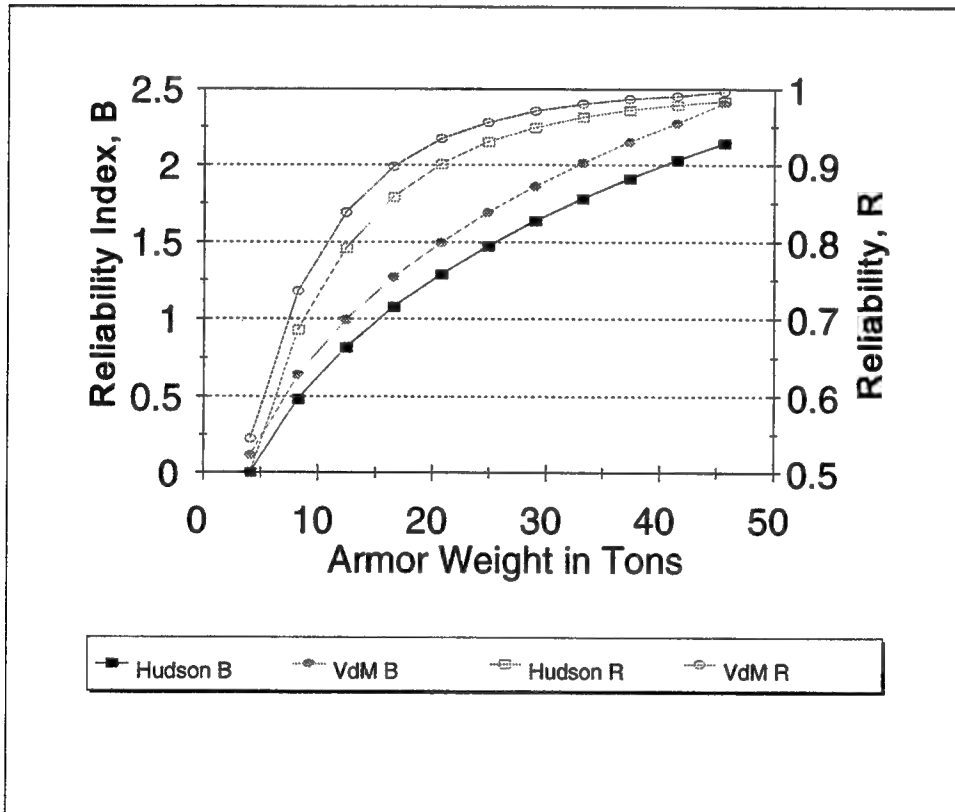


Figure 6. ITSFD solution for stone stability limit state

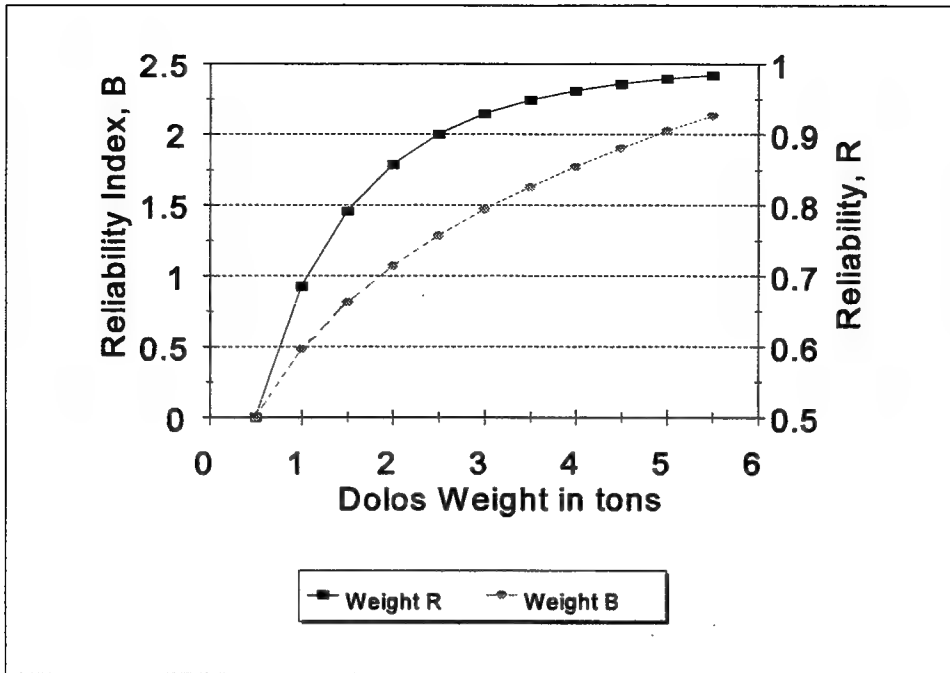


Figure 7. ITSPD solution for dolos stability limit state

TSFD reliability calculation

The underlying calculations for the TSFD safety factor approach in Equations 20 and 21 with the partial derivatives evaluated using the finite difference calculation of Equation 23 are shown in Table 3. The stone weight for this calculation corresponds to $K_D = 2$. It can be seen that the values of each variable are all mean values except for a single perturbation one standard deviation above the mean and one below. The reliability index is calculated as $\beta = \mu_{\ln F} / \sigma_{\ln F} = 1.4$. The influence factors, given by

$$\alpha_{x_i} = \frac{\partial F}{\partial x_i} \sigma_{x_i} \quad (29)$$

show the influence of each variable on the overall reliability. It is clear that, for this formulation of the Hudson performance function, wave height is dominant and that all other variables could be considered to be constants without significantly affecting the calculation. This is due to the fact that the wave height is cubed in the performance function, and is due to the broad wave height distribution. If a narrower wave height distribution were used, the influence of the stability coefficient could be similar to that of the wave height.

Table 3 TSFD Reliability Calculation for Hudson Limit State							
Armor Weight W kg	Structure Slope cot θ	Hudson Coeff. KD	Armor Specific Weight γ kg/m ³	Wave Height H m	Safety Factor F	Finite Difference $\partial F/\partial x$	Influence Factor $\alpha^2/\sum \alpha^2$ %
45013	2	2.59	2,405	2.70	10.97	0.1003	0.1
36829	2	2.59	2,405	2.70	10.75		
40921	2.1	2.59	2,405	2.70	11.40	0.0500	0.0
40921	1.9	2.59	2,405	2.70	10.32		
40921	2	3.24	2,405	2.70	13.58	0.2564	0.3
40921	2	1.94	2,405	2.70	8.13		
40921	2	2.59	2,477	2.70	12.18	0.1271	0.1
40921	2	2.59	2,333	2.70	9.62		
40921	2	2.59	2,405	4.11	3.08	-1.740	99.5
40921	2	2.59	2,405	1.25	99.78		

Comparison of reliability methods for Hudson performance function

Results from the Level II calculations, applied to the Hudson limit state, are summarized in Table 4 for a single stone armor weight and other parameters listed in Table 2. The stone weight of 41 tons utilized corresponds to $K_D = 2$, as the most conservative recommendation in the SPM. It is clear from Table 4 that the TSFD gives a more conservative result than the ITSFD method. The TSFD is simpler but accuracy is sacrificed. Note that reliability was computed using the ITSFD approach with two different versions of the Hudson equation performance function. The resulting reliability and reliability index were identical. Therefore, the ITSFD was verified to be invariant to the form of the limit state.

Table 4 Comparison of Taylor Series Approximations of Reliability for Hudson Limit State			
Reliability Computation Method	Reliability R	Reliability Index β	Difference in R From ITSFD %
ITSFD (Equation 3)	0.98	2.0	-
ITSFD (Equation 4)	0.98	2.0	-
TSFD	0.91	1.4	-7

Van der Meer Armor Stability Limit State

Armor stability equations for two-dimensional trunk sections were proposed by van der Meer (1987) for stone, cubes, Accropodes, and Tetrapods. Although not verified for general use in three-dimensional breakwater applications, these equations provide insight into various parameters of interest in stability, including wave period, porosity, degree of damage, and duration of storm, that are not included explicitly in the Hudson equation. The equations have been rearranged into limit state relations as follows:

STONE, $\xi \leq \xi_c$

$$g = K_{st} 6.2 \Delta D_n \sqrt{\cot \theta} S^{0.2} P^{0.18} S_m^{0.25} N^{-0.1} - H_s \quad (30)$$

STONE, $\xi > \xi_c$

$$g = K_{sg} 1.0 \Delta D_n \cot \theta^{(0.5-P)} S^{0.2} P^{-0.13} S_m^{-0.5P} N^{-0.1} - H_s \quad (31)$$

$$\xi_c = (6.2 P^{0.31} \sqrt{\tan \alpha})^{\frac{1}{P+0.5}} \quad (32)$$

where $\xi = \tan \theta / (S_m)^{0.5}$, $S_m = H_s / L_o$, $L_o = g T_z^2 / 2\pi$, T_z is the mean wave period, P is the structure porosity, S is the degree of damage, N is the number of waves, and K_{st} and K_{sg} are empirical coefficients.

CUBES

$$g = K_c \Delta D_n S_m^{-0.7} \left(6.7 \frac{N_o^{0.4}}{N^{0.3}} + 1.0 \right) - H_s \quad (33)$$

TETRAPODS

$$g = K_t \Delta D_n S_m^{-0.2} \left(3.75 \frac{N_o^{0.5}}{N^{0.25}} + 0.85 \right) - H_s \quad (34)$$

where N_0 is the zero-damage number of waves. Table 5 provides sample statistics for application of the various parameters in the equations listed above.

Figure 6 compares the ITSFD method applied to the van der Meer equations to that from the Hudson equation. For these input conditions, the two equations yield very similar reliability values for the range of armor weights. Comparing van der Meer equations with the Hudson results, it is clear that the van der Meer equations yield a higher reliability than does the Hudson equation.

Table 5 Statistics for Van der Meer Equation Limit State Variables			
Random Variable	Mean	Standard Deviation	Coefficient of Variation
	μ	σ	c
Stone nom. diameter, D_n	Range	Range	0.01
Stone stab. eq. coefficient, K_{st}	6.2	0.4	0.065
Stone stab. eq. coefficient, K_{sg}	1.0	0.08	0.08
Stone specific gravity, S_r	2.6	0.08	0.03
Tetrapod nom. diameter, D_n	Range	Range	0.0
Tetrapod eq. coefficient, K_t	1.0	0.1	0.10
Concrete spec. gravity, S_r	2.34	0.07	0.03
Cube nom. diameter, D_n	Range	Range	0.0
Cube eq. coefficient, K_c	1.0	0.1	0.10
Wave height, H	2.70 m	1.41 m	0.52
Wave steepness, S_m	0.04	0.01	0.25
Porosity, P	0.5	0.05	0.10
Number of waves, N	2000	600	0.03
Structure slope, $\cot\theta$	2.0	0.1	0.05

Concrete Armor Structural Limit State

The concrete armor unit strength design methods of Melby (1989, 1993) provide a Level III reliability-based analysis method where various loading dis-

tributions are convolved to determine a design load distribution. Here, the demand is characterized as a design tensile stress for the armor layer and the capacity is the tensile strength of the unreinforced concrete. A design probability of exceedance is specified in order to determine the design stress from the total stress probability distribution. This design stress is compared with a predefined capacity, a fatigue-reduced concrete strength. For this Level III analysis method, the design probability of exceedance E is used to determine a design stress from the maximum stress design probability distribution. This design exceedance is characterized as the probability of exceeding the design stress in the armor layer.

To relate the Level III methods to the Level II analysis method, the design exceedance is related to β through the basic reliability relation $E = 1 - R = 1 - \Phi(\beta)$. For concrete armor units, the limiting state of structural performance is given by

$$g = f'_t - \sigma_c = 0 \quad (35)$$

where f'_t is the flexural tensile strength and σ_c is the maximum principal tensile stress in the armor unit. Ellingwood et al. (1980) summarized a number of studies of the distribution of the tensile strength of concrete and, based on this work, a coefficient of variation of 18 percent is suggested (Mlakar 1995). The distribution of σ_c is computed in PC-ARMOR (Melby 1989, 1993) and used in the reliability analysis to determine the reliability with respect to armor strength.

An example calculation was done using PCARMOR to compute a mean of 3.05 MPa and a standard deviation of 1.46 MPa. The results of a simple analysis are shown in Figure 8.

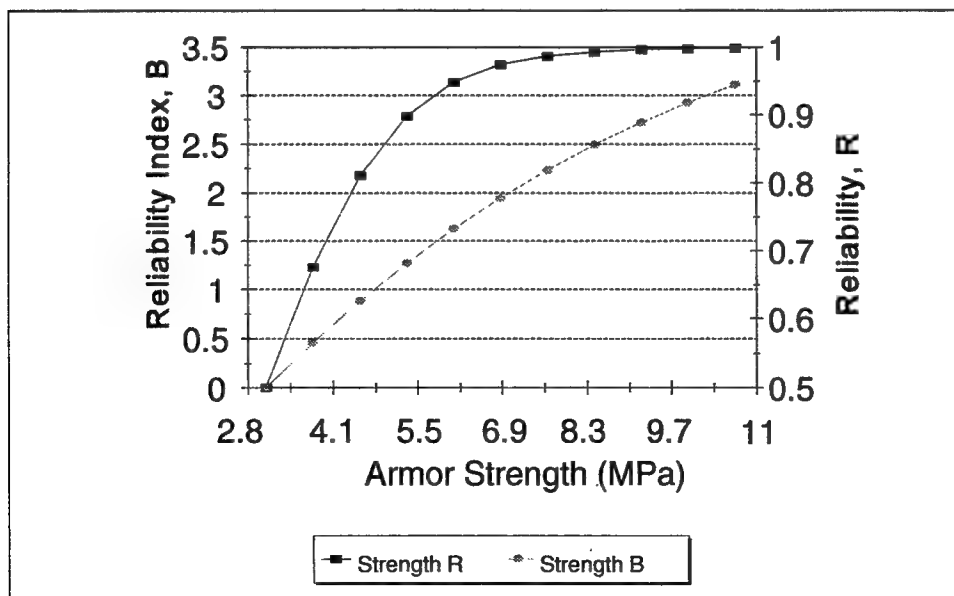


Figure 8. ITSFD solution for dolos structural limit state

Runup and Overtopping Limit States

As stated previously, the performance functions of SPM sections 7-II-1 and 7-II-2 are used for runup and overtopping, respectively. The equation for runup is given as

$$\frac{R}{H_s} = 1[1 - \exp(-\xi/2)] \quad (36)$$

where R is the significant runup, H_s the significant wave height, and ξ the surf similarity parameter as used previously. A safety margin limit state can be defined as the difference between the freeboard f and the runup

$$g = f - R = f - 1.1H[1 - \exp(-\xi/2)] \quad (37)$$

The safety factor limit state could be stated as $F = f/R$.

For overtopping, the equation given in the SPM is

$$K_{TO} = (0.51 - 0.11B/h)(1.0 - f/R) \quad (38)$$

where K_{TO} is the coefficient for transmission by overtopping, B the crest width, and h the crest height from the bottom. The safety margin limit state equation can be expressed as

$$g = H_{allow} - K_{TO}H \quad (39)$$

Both runup and overtopping have been incorporated into the ITSFD reliability methods and the safety margin relations encoded similar to the example given in Appendix A. Input statistics for the calculations are shown in Table 6. Results from an example computation are shown in Figures 9 and 10.

Table 6 Statistics for Runup and Overtopping Limit State Variables			
Random Variable	Mean	Standard Deviation	Coefficient of Variation
	μ	σ	cov %
Freeboard, f	Range	Range	5
Crest width, B	2.13 m	0.11 m	5
Crest height, h	3.66 m	0.18 m	5
Wave height, H	2.70 m	1.41 m	5
Wave period, T	10 sec	1 sec	10
Transmitted wave height, H_{allow}	range	range	5

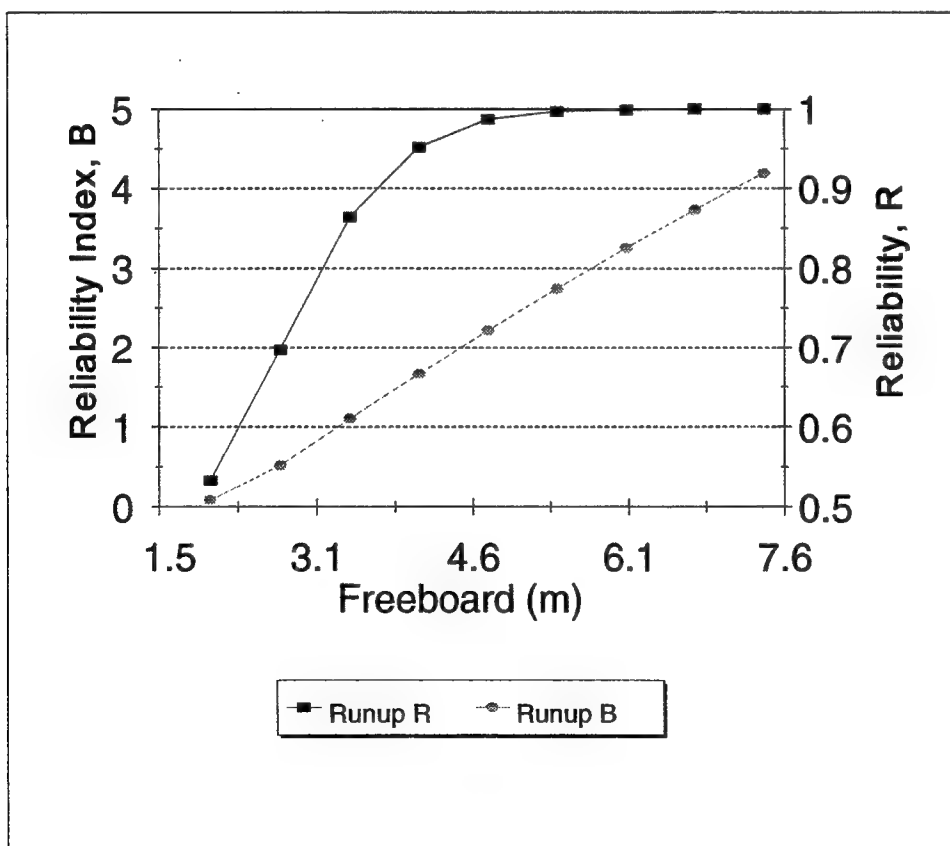


Figure 9. ITSFD solution for runup limit state

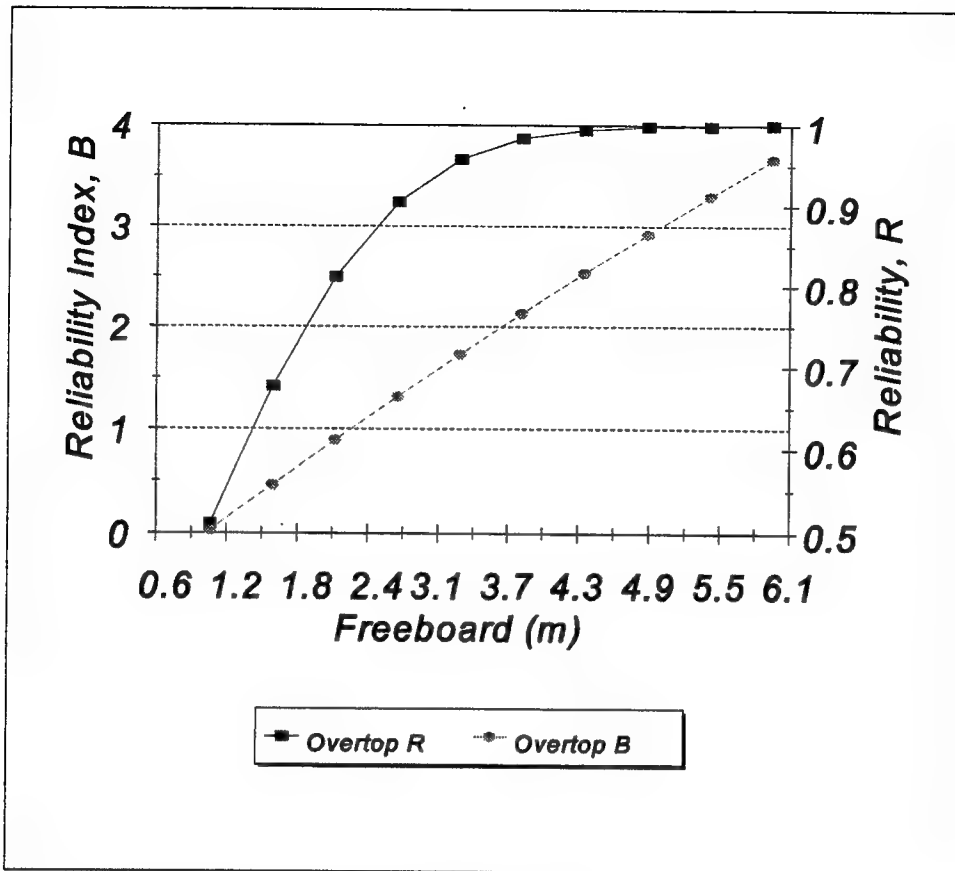


Figure 10. ITSFD solution for overtopping limit state

4 Conclusions

In this report, several level II reliability techniques are discussed and applied to breakwater design. Using these methods, the reliability and reliability index are determined for the dominant performance functions of a breakwater including stone and concrete armor stability, concrete armor structural response, runup, and overtopping. The methods include Taylor series finite difference (TSFD) methods, which are shown to be easily computed using a handheld calculator or spreadsheet. The TSFD method is shown to yield reasonable accuracy for preliminary comparison of various alternatives. An invariant iterative TSFD method and associated FORTRAN program are shown, which provide an improved approximation to the reliability for design and evaluation purposes.

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Appendix A Computer Pro- gram Listings

```

      PROGRAM RELIABLE
c
c Level II iterative invariant first order Taylor series design point
c reliability engine. Program computes reliability and reliability index
c for a range of independent variable values. The algorithm is invariant
c with respect to the form of the limit state equation.
c
c The limit state equation is computed in the subroutine LIMSTATE.
c The program computes various values of the independent variable in
c subroutine DEP_MEAN.
c
c Data are input from a user-edited file called R_INPUT.DAT and data are
c Output to a file called R_OUTPUT.DAT.
c
c Melby 11/95
c
c SCALAR DEFINITION
c      betaz              Reliability Index at z
c      betaz_p            Reliability Index at design point
c      g                  Limit state
c      gz                 Limit state at z
c      I,J                Indices for random variable
c      ik#                Iteration kounter #
c      ikl                Index
c      LAMBDA              Linear coefficient between z' and g
c      NumRV              Number of random variables
c      R                  Reliability
c      cov                std / mean
c      zd                 z at design point
c
c VECTOR DEFINITION
c      grad_g(NumRV)      Gradient of limit state
c      Mean(NumRV),m      Means
c      Std(NumRV),s       Standard devs
c      X(NumRV)           Random variables
c      z(NumRV)           Standard normal random variables
c      zp(NumRV)          Design point
c
c *****
c      PARAMETER (NRV=10)
c      REAL mean(NRV), Std(NRV), z(NRV), zp(NRV), grad_g(NRV)
c      REAL Const(NRV), cov(NRV), x(NRV), lambda
c      CHARACTER textc(NRV)*15, textR(NRV)*15, header*60, texth*15
c
c -----
c      Open I/O files
c -----
c      OPEN(5, FILE='R_input.dat', status='old')
c      OPEN(2, FILE='R_output.dat', status='unknown')
c
c -----
c      Read input data file
c -----
c      PRINT*, ' Program will now read input file named R_input.dat'
c      PRINT*
c      PRINT*, ' Input statistics for each variable are input in this file. For each random'
c      PRINT*, ' variable, the mean, std. Dev., and coef. of variation are read. Only the mean'
c      PRINT*, ' and s.d. are required for each variable except the last one input, which'
c      PRINT*, ' will be treated as the dependent variable and the reliability computed over a'
c      PRINT*, ' range of values.'
c      PRINT*
c      PAUSE
c      READ(1, '(A60)') header
c      PRINT*, header
c      READ(1, '(A60)') header
c      PRINT*, header
c      READ(1, *) texth, numRV, NumConst, NumDepVar

```



```

PRINT*
PRINT*, '----- Values read from input file -----'
PRINT*, 'Number of random variables', NumRV
PRINT*, 'Number of constants', NumConst
PRINT*, 'Number of variable calculations', NumDepVar
PRINT*
READ(1, '(A60)') header
PRINT*, '----- Random variable statistics read from input file -----'
PRINT*, 'Param      Mean      Std Dev   Coef of Var'
PRINT*, '-----'
DO 5, ik0 = 1, NumRV
  READ(1, *) textrv(ik0), mean(ik0), Std(ik0), Cov(ik0)
  PRINT*, textrv(ik0), mean(ik0), Std(ik0), Cov(ik0)
5 CONTINUE
READ(1, '(A60)')
PRINT*
PRINT*, '----- Constants read from input file -----'
PRINT*, 'Constant      Value'
PRINT*, '-----'
DO 6, ik0 = 1, NumConst
  READ(1, *) textc(ik0), Const(ik0)
  PRINT*, textc(ik0), Const(ik0)
6 CONTINUE
PRINT*
PRINT*, 'Values OK? If not, press CTRL-C and modify the input file'
PAUSE

c
c-----
c Write output file headers
c-----
  WRITE(2, *) 'Output data file from RELIABLE'
  WRITE(2, *)
  WRITE(2, *) '----- Constants read from input file -----'
  WRITE(2, *) 'Number of random variables, constants, and dependent var. calcs.'
  WRITE(2, *) '-----'
  WRITE(2, *) NumRV, NumConst, NumDepVar
  WRITE(2, *)
  WRITE(2, *) '----- Parameters read from input file -----'
  WRITE(2, *) texth = 'texth,' NumRV = ' numRV, ' NumConst = ' NumConst
  WRITE(2, *)
  WRITE(2, *) Parameter              Mean              Std. Dev. Coef. of Var.'
  WRITE(2, *) '-----'
  DO 7 ik0 = 1, NumRV
    WRITE(2, *) textrv(ik0), mean(ik0), Std(ik0), Cov(ik0)
7 CONTINUE
  WRITE(2, *)
  WRITE(2, *) 'Constant      Value'
  WRITE(2, *) '-----'
  DO 8 ik0 = 1, NumConst
    WRITE(2, *) textc(ik0), Const(ik0)
8 CONTINUE
  WRITE(2, *)
  WRITE(2, *) '----- Output Results -----'
  WRITE(2, *)
  WRITE(2, *) Mean      Beta      R'
  WRITE(2, *) '-----'

c
c-----
c Initialize variables
c-----
  DO 10 kount = 1, NumRV-1
    z(kount) = 0.0
10 CONTINUE
c

```

```

c-----
c Begin Computations
c-----
      DO 20 ikl=1, NumDepVar
      CALL DEP_MEAN(ikl, mean, NRV, Const, NumRV)
      Std(NumRV) = cov(NumRV) * Mean(NumRV)
      Z(NumRV) = 0.0
      betaz = 0.0
30    CONTINUE

      CALL LIMSTATE(z, x, std, mean, Const, g, NRV, NumRV)
      gz = g
      grad_g0 = 0.0
      zd = 0.0
c
      DO 40 ik2 = 1, NumRV
      z(ik2) = z(ik2) + 0.001
      CALL LIMSTATE(z, x, std, mean, Const, g, NRV, NumRV)
      z(ik2) = z(ik2) - 0.001
      grad_g(ik2) = (g - gz)/0.001
      grad_g0 = grad_g0 + grad_g(ik2)*grad_g(ik2)
      zd = zd + z(ik2) * grad_g(ik2)
40    CONTINUE
c
      lambda = (zd - gz) / grad_g0
c
c Find zp and betaz_P
      betaz_P = 0.0
      DO 50 ik3 = 1, NumRV
      zp(ik3) = lambda * grad_g(ik3)
      betaz_P = betaz_P + zp(ik3) * zp(ik3)
50    CONTINUE
      betaz_P = SQRT(betaz_P)
      IF(ABS(betaz_P - betaz) .LT. 0.001) GOTO 90
      betaz = betaz_P
c
      DO 60 ik4 = 1, NumRV
      z(ik4) = zp(ik4)
60    CONTINUE
      GOTO 30
90    CONTINUE ! solution converged
c
c Hastings rational approximation for normal reliability
      R = 1. - 0.5/(1. + 0.196854 *betaz_P +0.115194*betaz_P*betaz_P+
&      0.000344*betaz_P**3 + 0.019527*betaz_P**4)**4
c
      WRITE(2,100) mean(NumRV), betaz_P, R
100   FORMAT(1x, 3(F15.3))
20    CONTINUE
      CLOSE(1)
      CLOSE(2)
      STOP
      END

```

```

      SUBROUTINE limstate(z, x, s, m, C, g, NRV, NumRV)
      REAL z(NRV), x(NRV), s(NRV), m(NRV), C(NRV)
c Hudson Stability Equation
c Random Variables, mean, standard deviation
c Concr Specific Wt      Gamma_r      m(1)      s(1)
c Structure Slope        cot_theta     m(2)      s(2)
c Coefficient            KD             m(3)      s(3)
c Wave Height            H              m(4)      s(4)
c Armor Weight           W              m(5)      s(5)
c
c Constants
c H2O Specific Weight    Gamma_w      c(1)
c
      DO 10 j=1,NumRV
        x(j) = z(j) * s(j) + m(j)
10    CONTINUE
      Delta = x(1)/C(1) - 1.
      g = x(2) * x(3) * Delta**3 * x(5) - x(1) * x(4)**3
      RETURN
      END

      SUBROUTINE dep_mean(kount, m, NRV, C, NumRV)
      REAL m(NRV), C(NRV)
c Hudson Stability Equation
c Random Variables, mean, standard deviation
c Concr Specific Wt      Gamma_r      m(1)      s(1)
c Structure Slope        cot_theta     m(2)      s(2)
c Coefficient            KD             m(3)      s(3)
c Wave Height            H              m(4)      s(4)
c Armor Weight           W              m(5)      s(5)
c
c Constants
c H2O Specific Weight    Gamma_w      c(1)
c
      Delta = m(1)/C(1) - 1
      m(NumRV) = FLOAT(kount) * m(1) * (m(4)/Delta)**3 / m(3) / m(2)
      RETURN
      END

```

Hudson Damage Function Reliability Input in File R_INPUT.DAT

Sample Stone Stability Run, Melby, 19 Nov 1995

Constants 5 1 11

Input Random Variable Statistics

Gamma_r	2405.0	72.0	-1.
cot	2.	0.1	-1.
KD	2.59	0.65	-1.
H	2.70	1.41	-1.
W	-1.0	-1.0	0.2

Input Constants in Limit State Equation

Gamma_w 1024.0

Output data file from RELIABLE

----- Constants read from input file -----

Number of random variables, constants, and dependent var. calcs.

5 1 11

----- Params read from input file -----

texth = Constants NumRV = 5 NumConst = 1

Parameter	Mean	Std Dev	Coef of Var
Gamma_r	2405.0	72.000	-1.0000
cot	2.0000	0.1000	-1.0000
KD	2.5900	0.6500	-1.0000
H	2.7000	1.4100	-1.0000
W	-1.000	-1.000	0.20000

Constant	Value
----------	-------

Gamma_w	1024.0
---------	--------

----- Output Results -----

Mean	Beta	R
3768.453	0.000	0.500
7536.913	0.483	0.686
11305.357	0.815	0.792
15073.811	1.074	0.858
18842.263	1.289	0.901
22610.715	1.473	0.930
26379.168	1.635	0.949
30147.622	1.780	0.963
33916.074	1.912	0.972
37684.526	2.032	0.979
41452.977	2.142	0.984

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